

The contents of handling data and geometry in the early years of new mathematics curriculum in Slovenia

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Abstract

In the article, the authors present and establish basic changes of mathematical education at the beginning of a 9-year primary school in Slovenia. The new mathematics curriculum in Slovenia brings the following main changes: extension of a mathematical problem, introduction of handling data in the early years of elementary school, and according to different models of abstractions, a new concept of learning geometry called ‘geometry from the figure to the point’. The authors also stress that the main intention of the new mathematical syllabus is not only to teach pupils mathematics but also to give them opportunity to discover mathematics by themselves, and to think and extend their knowledge.

KEY WORDS: a 9-year primary school, curriculum, teaching mathematics, handling data, geometry.

1 Introduction

The mathematical curriculum for a 9-year primary school from 1998 entails up some very important changes at the beginning of education. We are going to present them in this article. The concept of teaching mathematics at the beginning of primary school, up to now based on the mathematical curriculum from 1983, has above all stressed confident abilities of performing mathematical algorithms and less the problem solving knowledge. Even if we do not analyse teaching mathematics in an 8-year primary school in detail, we may state that the Slovenian concept of teaching mathematics at the beginning of an 8-year primary school has not been kept up with the contemporary recognitions – pedagogical and didactical – of mathematical theories and, at the same time, it does not link up mathematics with children’s experiences or life. Children are exposed to mathematics every day. They arrange, classify, count, meet different shapes, measure, orientate themselves in the space and do simple mathematical operations. And they do all this in their special way. A three-year-old child may count one, two, three, five, seven, eleven, ... and s/he uses this order again and again when counting elements. A child has acquired a principle that the natural numbers are in order, but

s/he makes up her/his own order. A child does not deal with differences between a square and a cube or a triangle and a prism, etc. S/he uses mathematics in her/his own way, and does not forget about her/his own interests.

If not earlier, we want to teach children mathematics at school, and when teaching them, we consider their maturity, cognitive development, experiences, and pre-knowledge, their wrong images and, finally, their needs. When a child enters a nine-year elementary school, s/he is a year younger, and this is certainly a great challenge for organisers of teaching situations, i.e. a teacher and an educator. Besides other subjects, we start teaching her/him mathematics when s/he is still most interested in a game, when s/he is not emotionally mature yet, and does not divide the world into mathematical, sociological, etc., when s/he has problems with long-term concentration, and when s/he is only six years old. It is very important that the curriculum from 1998, on which teaching mathematics in the beginning of a 9-year primary school is based, follows contemporary pedagogical, psychological and didactic- mathematical theories. In the first three years of schooling, children are exposed to the mathematical alphabet, mathematical strategies and mathematical ideas. They pass from a pre-operational level to a concrete one, which means they have to gain their knowledge mostly on a concrete-experience level. Therefore enough didactic material has to be available, like games and toys. Only when we spend enough time on the concrete level, we pass to the iconic, and then to the symbolic one. When determining the way of learning mathematics, we deal with the teaching methods which lead pupils towards the appropriate knowledge. In the first few years of learning mathematics it is necessary to point out a game, an experience learning, observation and discussion. We choose a teaching method according to the mathematical content, pupils' knowledge and abilities, and finally, according to the available materials.

In Slovenia, there has lately been more and more discussions about problem solving, especially of a teaching method role where a teacher is mostly an organiser of mathematical challenges, which gives pupils an opportunity to gain conceptual as well as procedural knowledge, and meet mathematics as a useful and a nice experience.

At the beginning of education, all basic mathematical ideas should be built on concrete problem situations deriving from children's real experiences. To put children in a concrete problem situation does not always mean it has to come from a real everyday life. Children's reality is also their world of fantasy and imagination (Tenuta, 1992). However teaching mathematics through problem situations will not be easier, on the contrary, it will be even more difficult. But we believe that teaching by means of understanding problem situations is certainly one of the distinctive characteristics of a good teaching programme.

A 9-year primary school is therefore a great challenge. It is a challenge because of the described specifics of a six-year-old child, a different rule of mathematics in our lives, and because knowledge of Slovene pupils is comparable with knowledge of pupils from other countries. Some international researches (e.g. IAEP research (1991) and School Children's Acquisition and Maintenance of Quantitative Thinking in Mathematics research (1996)) have proved that mathematical knowledge of our pupils has not been balanced. As we have already said, Slovene pupils can calculate skilfully and well, but they have a lack of problem knowledge and knowledge of handling data in comparison with pupils from other participating countries. This is why we have introduced the following novelties into the mathematical curriculum from 1998:

- the extension of a mathematical problem,
- the content of handling data,
- a new approach to learning and teaching geometry.

Up to now, the calculating skills have been stressed. These skills are certainly crucial. They present an important tool without which mathematical thought is unproductive, but they are not the most important elements of teaching mathematics. The point is that pupils learn these skills with understanding, and that they are capable of using them in situations where this knowledge is required. These are mostly different mathematical problems which do not occur immediately after dealing with a certain calculating operation (as in Slovene mathematical teaching practice), but they are taken from a real or an imaginary world of children or adults, excel in the reality of problem solving procedures. As we know, data in the problems can be alternatives, there are too much data or too little, etc., and this demands a considerable reflection on the procedure or the strategy of solving the problem which has to be flexible and can be adapted to different situations. So it is definitely not mechanical.

The second very important novelty, introduced with a 9-year primary school in Slovenia, is the introduction of the content of handling data it will be explained in detail. Our pupils have not been systematically exposed to the handling data content, combinatorial situations and probability yet. The intention of these contents is to develop mathematical thought (specially with probability and combinatorics which are specific disciplines and demand a different way of thinking which is undetermined, creative and systematic as well), and to give pupils mathematical literacy. Pupils collect data in different ways, demonstrate and interpret them and, in this way, they acquire the skills which are necessary in today's world. Pupils also create their critical attitude to information presented in various media.

Geometry presents a special place in the new curriculum as well. Geometry which we call 'geometry from a figure to a point' will differ from the typical geometry taught in a 8-year primary school called 'geometry from a point to a figure' in the approach to creating the basic geometrical ideas. Teaching geometry will be based on geometrical figures, and the idea of various dimensions will be built gradually.

We will look at handling data and geometry in the early years of teaching and learning mathematics of a 9-year primary school in Slovenia in detail.

2 Handling data (the bases of statistics, combinatorics and probability)

The introduction of the content of handling data into a mathematics curriculum for primary schools (1998) presents one of the most important for a 9-year primary school. Handling data (statistics, combinatorics, probability) has been taught quite late (in secondary school) and only on a formal level. The main reasons for introducing these issues into the mathematics curriculum for primary school are the following:

- calculating literacy (tables, diagrams, questionnaires are parts of every day life (newspapers, student's textbooks, computerised presentation of data, etc.));
- needs for knowing the means of communication (today graphical surveys, tables, etc. are regularly used for communication);
- needs for the ability of critical judgement of presented data (if we do not understand the techniques of presenting data, and if we are not capable of judging them critically, we can be very easily manipulated (advertisements, elections, etc.));
- accessibility of calculation for handling data;
- incompatibility between our curriculum and curriculum of most European countries.

As we have already written, these issues have been introduced into a 9-year primary school at the beginning of learning mathematics (first triad). However in the first few years it is not a 'real' teaching of handling data. Children gain knowledge only intuitively; i.e. on concrete experienced level, and we prepare them for abstract thinking gradually.

Handling data at the early stage of learning certainly has its special purpose. It links mathematics with other subjects (Slovene language, social studies, science, etc.) and, in this way, creates the image of the whole teaching process (Cotič, Hodnik, 1995). Moreover, it develops the mathematical thinking, literacy and undetermined scheme of thinking or, as Cockcroft (1982) has written, these issues do not have only technical nature (learning different methods of handling data, systematic presentation of data, etc.) but they are also

distinguished for uncertainty and changeability. When we research uncertain and changeable data, they enable us to decide on our own.

Handling data includes issues from combinatorics, probability and statistics. The reasons for introducing them are the follow.

2.1 Reasons for teaching statistics

Fujita (1985) claims that the purpose of mathematical education is double: mathematical literacy and the ability of mathematical thinking. The development of this ability is certainly a complex activity, and is intended mostly (but not exclusively) for the pupils who will proceed with mathematical- science studies. However, it is not the same with mathematical literacy since it has to be the objective for all pupils in the elementary school. And statistics is mainly a part of 'mathematical literacy' (Fujita, 1985). At the beginning of schooling, statistics should be only the introduction into data presentation and analysis. So it presents the activity which is urgent in the world full of information. The issues from statistics have to be presented to pupils on experienced level already at the elementary stage of education because they gradually and slowly help pupils to develop critical attitude toward 'numeric information' presented by media (Howson, Kahane, 1986). Statistics also enables the links between other mathematical contents (arithmetic, geometry, etc.) and non-mathematical fields (natural science, social studies, Slovene language) and gives us the opportunity to gain the skills for graphical presentations in mathematics as well as at other fields.

2.2 Reasons for introducing combinatorics

It is true that statistics includes elements of mathematical thinking but it is certainly not the main stimulus of mathematical thinking for pupils (Fujita, 1985). Teaching combinatorics is mainly intended for developing systematic attitude to solving combinatorial situations and the inductive and analogical inference. When introducing combinatorics into a lower stage of primary school, a few dilemmas appear whether the children at this stage are able to solve this kind of problems. According to Piaget and Inhelder (1951), pupils are capable of solving them just on the level of formal operations (11-15 years). But their conclusions that combinatorics is not appropriate at this stage are based just on children's spontaneous answers without introducing these issues into the syllabus before. It has to be emphasised that, in the first years of education, we can not speak about 'real' combinatorics because it - as one of mathematical contents - demands methods that in combinatorial situations give us possibility

to determine the number of elements in a set without direct counting. Children simply count the elements in simple combinatorial situations.

Mathematical- didactic researches in the world on the appropriateness of introducing combinatorics into the lower primary school have disapproved Piaget's and Inhelder's statement. We would like to point out two studies done by Fishbein in Israel in 1970 and Cotič in Slovenia in 1998. They both introduced the issue in a way that is suitable for children of this age. Children start solving combinatorial situation from their experience (a game) on a concrete level; it means that they manipulate objects (but the number of them should be small) (Fishbein, 1975). They should use the objects from their everyday life, and they should perform appropriate activities. For example:

- they make necklaces (as many as it is possible) using wooden balls of various sizes,
- they arrange models of geometrical figures, cubes, etc. of various in every possible way, etc.

2.3 Reasons for teaching probability

What about the contents of probability? UNESCO (1972) has written that determinate thinking is not enough for understanding some sciences; indeterminate schemes of thinking are present and more and more necessary, e.g. in genetics, biology, physics, economy, etc. Today, probability is used even in the fields which are close to people's everyday life; meteorology, elections, insurance, etc.

The world we live in has been changing very fast. We are often exposed to new and unpredictable situations. Fishbein (1985) says that we live in the world full of uncertainty and unpredictability, and that we have to prepare our children for their future life so that they will be able to interpret it critically and to act consciously. That is why the 'alphabet' of probability is needed. It demands a special way of thinking which is different from a determinate way predominating in our schools. Adults who were exposed to such education often have problems to understand the basic and crucial probability ideas because the bivalent logic (wrong/correct, true/false) fails. Fishbein did the research on the appropriateness of probability in the lower primary school already in 1970. In Slovenia, Cotič did a similar research in 1998. They have both concluded that children in lower primary school accept and learn the basic concepts of probability on intuitive and experienced level well. After his research, Fishbein (1985) has written that they have proved their thesis that the probability concepts and techniques had to be introduced already in lower primary school and not only in

secondary school or even in vocational school when the pupils' attitude was already formed. He also stated that if we wanted pupils to develop their attitude which considerably differed from the determinate scheme and attitude, we had to start teaching statistics and probability on the level of concrete operations (7-11 years), if not even earlier, or not later than in the phase when the pupils proceeded from this level of concrete operations to the level of formal operations (10-12 years).

3 Geometry

Geometry has an important place in mathematics because it, as Usiskin (1990) adduces:

- enables the study of the physical world,
- deals with visualization, drawing and construction of the figures,
- enables the representation of the concepts in mathematics which are not visual,
- it is an example of a mathematical system,
- it gives us pleasure and it is esthetical, added Willson (1977).

The school geometry usually does not satisfy all these criteria, especially not the last one. To meet the last criteria, especially in Great Britain and in the USA, some elements of 'non-Euclidean' geometry can be noticed, which should make geometry more attractive to pupils. Mobius' strip, four colours conjecture, tessellation (similar to Escher's presentation), fractals, some kinds of intuitive presentations of 'non-Euclidean' geometry like geometry on a ball, a spoon, etc. can be found in pupils' textbooks. These ideas are isolated and we do not broaden them systematically, because it would be too difficult for children in primary school. These children, who will later be occupied with mathematics much, have some opportunities to look into dimensions of mathematics that they would otherwise never meet.

Geometry in our schools may be defined as teaching of geometrical ideas according to the 'step by step' principle. The structure of geometrical knowledge adapted for children in primary school which children gain systematically is mainly considered here. We have to mention that the abstraction process of the ideas in geometry is different from that one in arithmetic. Piaget talks about three forms of abstraction. When the objects are prominent and their features are important for the construction of the knowledge, he talks about the empirical abstraction. When the processes are prominent, we talk about the pseudoempirical abstraction that contains features of processes with objects (Piaget, 1985). The third form is the reflexive abstraction which, connected to the individual's cognitive structure, leads to the construction of new structures according to the reflection of thoughts or reflection of previous abstractions.

We could come to a conclusion that two abstractions exist; one referring to the objects and the other to the processes. When building the ideas in geometry, we could talk about the abstraction referring to the objects and, when building ideas in arithmetic, it is the abstraction referring to the processes with objects (Gray, Pinto, Pitta, Tall, 1999).

Processes or strategies could, in a certain phase, become constituent elements of a new object (Cottrill and others, 1996). When a child makes her/his first step into the empirical or pseudoempirical abstraction, s/he constructs certain mental steps which become constituent parts of all further processes, and lead to the hierarchy of mental constructions or concepts (Gray, Pinto, Pitta, Tall, 1999). We could also say that the concept of a number starts with the pseudoempirical abstraction, and the concept in geometry with the empirical abstraction. At the empirical abstraction pupils learn the features of objects through physical experiences (for example: a cylinder rolls which means that it is curved). In the process of the pseudoempirical abstraction pupils are focused more on a process (way) of handling with objects than on objects (when counting balls, a child is focused on the process of counting and not on balls).

In the process of forming concepts in geometry we must not ignore the role of language. The perception of forms is crucial in geometry, and we can make hierarchy of the concepts in geometry only by means of a language. We perceive the forms as *gestalts*¹ first and then we classify and name them: a triangle, a square, a rectangle, etc. By means of a verbal discussion and construction a pupil will be able to learn the hierarchy among the geometrical shapes: a square is a rectangle or a square is a rhomb, and a parallelogram is a four-angled shape. The perception and description of geometrical forms lead to the mental construction of the objects and to the development of concepts in Euclidean geometry.

Van Hiele defined the method where the observing features of figures is essential and leads to forming concepts in geometry. We are going to present it briefly. Van Hiele (1986) defined three levels of gaining the geometrical knowledge: a visual level, a descriptive level and a theoretical one.

¹ The beginners of entirety-orientated tradition of teaching are a group of gestalt psychologists (Ger. *gestalt*: a sign, a figure). They have interpreted the individual's experiences as an organized whole and not as a sum of individual units which form the experience or experiences. Their attitude is that it is less possible pupils will understand the problem if the teacher gives them the answer. They think that pupils should solve the problem by themselves with their own activity. Stern has adopted this theory and strived for using structural material at schools. We do not claim that she was the first who suggested the usage of the structural materials at schools, but she is certainly the first who established the need for introducing the structural material into the teaching of mathematics from the theoretical point of view. The leading representative of gestalt psychologists in the mathematical field was Wertheimer.

The important phases in each level are:

providing the information, directed orientation, free observation, explication and integration.

Theoretical level	<i>Use deductive reasoning to prove geometric relationship</i>
The process of learning	The phases of learning Integration Free orientation Explication Directed orientation Information
Descriptive level	<i>Recognise objects by their geometric properties</i>
The process of learning	The phases of learning Integration Free orientation Explication Directed orientation Information
Visual level	<i>Recognise geometric objects globally</i>

Table 1: Van Hiel's model of teaching geometry (Teppo, 1991, p.210)

In Slovenia, teaching geometry has also been treated according to the Van Hiel's model. If we join the empirical abstraction and visual stage, which is the starting stage of teaching forms, it is obvious that pupils have to base their knowledge on concrete objects or three-dimensional ones. It is necessary to start with three-dimensional world and then pass to smaller dimensions gradually.

In an 8-year primary school, the Euclidean geometry is built from the smallest dimension, a point, to the three-dimensional objects. So we follow mathematical structure more than a child. This aspect is called a 'geometry from a point to a figure'. Pupils become familiar with the forms first, e.g: a triangular, a square, a circular, and then they meet different kinds of lines. At the end of the first grade they gain the idea, that a point is an intersection of two lines. Here two things are questionable. First, how to present the situation to children mathematically correctly and how this idea meets the children's world as it is the world of concrete things. We agree that each presentation of mathematical ideas is just approximate, but some are closer to it than the others. The illustration of straight lines with a straw and an intersection with a small lump of plasticine is certainly a part of the later. We would also like

to mention the research done by Fischbein (1993) on how 6 to 11-year-old children understand the idea of a point. He asked children to compare point A with point B where point A is an intersection of four lines and point B an intersection of two lines. He asked pupils the following questions: Are these two points various? Is one of them bigger? Are the points the same shape? Most of six-year-old children (68%) did not answer any of the questions because they did not understand the presentation of a point. Eleven-year-old children answered the questions as follows: 20% of them answered that point A is bigger than point B, and 45% answered that they are the same.

Alternative attitude 'a geometry from a figure to a point' is a novelty in a 9-year primary school in Slovenia. According to this attitude, children meet and look for three-dimensional shapes in their surrounding first, then they search for objects which are similar to the models of geometrical figures, meet their features, and finally they make models of geometrical figures from different materials. Proceeding to two-dimensional shapes can also be enabled by printing sides of 3-D geometrical shapes in clay, sand, or as a stamp on a paper. We have to point out that we do not get 2-D shapes (just in case of stamping sides of 3-D shapes on paper), but different figures (round, triangular, etc.). This attitude enables easier transition from a preschool to a school period, and it fits mathematical criteria, and is more understandable for children. At some points, teaching geometry will be identical to teaching geometry in an 8-year primary school again because it is practically impossible to build structures in geometry without introducing the point. We can conclude that with slightly different attitude in geometry we will still satisfy the criteria of mathematical society for teaching geometry as a mathematical content and, at the same time, will bring geometry closer to children and help them to overcome some difficulties which they come across when learning geometrical ideas.

5 Conclusion

The new mathematical curriculum in Slovenia from 1998 does not allow mechanistic teaching of ideas, rules and operations. Its crucial aim is a different role of pupils. They should become active modellers of their own learning. Solving problems is characteristic for mathematical teaching besides gaining mathematical ideas, rules and operations. Different mathematical ideas and theories were developed in situations when the known means and ways of solving the problems were not sufficient. Problems are usually the result of needs, intellectual interest or curiosity. Pupils will strive to understand the mathematical idea if they are interested in the

problem (Cotič, Felda, 2001). This is the reason why the new syllabus introduces harmoniousness of solving problems and pupils' readiness for asking questions and looking for answers. However, that it is not going to happen by itself. Only a well-qualified teacher is capable of developing and encouraging such learning, listening to pupils' needs, motivating, and preparing them a corresponding problem or a teaching situation. We have to train students and primary school teachers for teaching mathematics at the beginning of education in a 9-year primary school. Education, which has been provided up to now, stresses only the teaching in the first grade of a primary school, and too little stress is given to the teaching in higher grades. Difficulties which some teachers who teach in the second or third grade of a 9-year primary school have, indicate that they transmit the model of teaching from an 8-year primary school to a 9-year primary school. The 8-year primary school model does not stress flexible movement from one representative system to another (e.g. from enactive to iconic), and a symbolic level is introduced too fast. Moreover, mathematical ideas are not formed through problem situations. There is a danger that mathematical lessons in the beginning of a primary school could again become training of calculations and learning of mathematical facts by heart.

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